# **Module 2 – Rational Numbers**

# **Lessons 17-23**

# **Problem Set**

# Lesson 17 Problem Set

# Lesson Summary

Tape Diagrams can be used to model and identify the sequence of operations to find a solution algebraically.

The goal in solving equations algebraically is to isolate the variable.

The process of doing this requires "undoing" addition or subtraction to obtain a 0 and "undoing" multiplication or division to obtain a 1. The additive inverse and multiplicative inverse properties are applied, to get the 0 (the additive identity) and 1 (the multiplicative identity).

The addition and multiplication properties of equality are applied because in an equation, A = B, when a number is added or multiplied to both sides, the resulting sum or product remains equal.

- 1. A taxi cab in Myrtle Beach charges \$2 per mile and \$1 for every person. If a taxi cab ride for two people costs \$12, how far did the taxi cab travel?
- 2. Heather works as a waitress at her family's restaurant. She works 2 hr. every morning during the breakfast shift and the same number of hours every evening during the dinner shift. In the last four days she worked 28 hr. How many hours did she work during each dinner shift?
- 3. Jillian exercises 5 times a week. She runs 3 mi. each morning and bikes in the evening. If she exercises a total of 30 miles for the week, how many miles does she bike each evening?
- 4. Marc eats an egg sandwich for breakfast and a big burger for lunch every day. The egg sandwich has 250 cal. If Marc has 5,250 cal. for breakfast and lunch for the week in total, how many calories are in one big burger?
- 5. Jackie won tickets playing the bowling game at the local arcade. The first time, she won 60 tickets. The second time, she won a bonus, which was 4 times the number of tickets of the original second prize. All together she won 200 tickets. How many tickets was the original second prize?

# Lesson 18 Problem Set

#### Lesson Summary

- An expression is a number or a letter, which can be raised to a whole number exponent. An expression can be a product whose factors are any one of the entities described above. An expression can also be the sum and/or difference of the products described above.
- To evaluate an expression, replace each variable with its corresponding numerical value. Using order of operations, the expression can be written as a single numerical value.
- Expressions are equivalent if they evaluate to the same number for every substitution of numbers into all the letters in each expression.
  - Sally is paid a fixed amount of money to walk her neighbor's dog every day after school. Each month, when she is paid, she puts aside \$20 to spend and saves the remaining amount. Write an expression that represents the amount Sally will save in 6 months if she earns *m* dollars each month. If Sally is paid \$65 each month, how much will she save in 6 months?
- 2. A football team scored 3 touchdowns, 3 extra points and 4 field goals. Write an expression to represent the total points the football team scored.

Write another expression that is equivalent to the one written above.

If each touchdown is worth 6 points, each extra point is 1 point, and each field goal is 3 points, how many total points did the team score?

3. Write three other expressions that are equivalent to 8x - 12.

Name	Date
	Bate

Profit is defined as earnings less expenses (earnings – expenses). At the local hot air balloon festival, the Ma & Pops Ice Cream Truck sells ice cream pops, which cost them \$0.75 each, for \$2 each. They also paid \$50 to the festival's organizers for a vendor permit. The table below shows the earnings, expenses and profit earned when 50, 75 and 100 ice cream pops were sold at the festival.

Number of Pops Sold	Earnings	Expenses	Profit
50	50(2) - 100	50(0.75) + 50	100 - 87.5
50	30(2) = 100	37.5 + 50 = 87.5	12.50
75	75(2) = 150	75(0.75) + 50 56.25 + 50 = 106.25	150 - 106.25 = 43.75
100	100(2) = 200	100(0.75) + 50 75 + 50 = 125	200 - 125 = 75

Write an expression that represents the profit (in dollars) Ma & Pop earned by selling ice cream pops at the festival.

Write an equivalent expression.

How much did Ma & Pops Ice Cream Truck profit if it sold 20 ice cream pops? What does this mean? Explain why this might be the case?

6. How much did Ma & Pops Ice Cream truck profit if it sold 75 Ice Cream Pops? What does this mean? Explain why this might be the case?

# Lesson 19 Problem Set

# Lesson Summary

- Two expressions are equivalent if they yield the same number for every substitution of numbers for the letters in each expression.
- The expression that allows us to find the cost of an item after the discount has been taken and the sales tax has been added is written by representing the discount price added to the discount price multiplied by the sales tax rate.

1. A family of 12 went to the local Italian restaurant for dinner. Every family member ordered a drink and meal, 3 ordered an appetizer, and 6 people ordered cake for dessert.

- a. Write an expression that can be used to figure out the cost of the bill. Include the definitions for the variables the server used.
- b. The waitress wrote on her ordering pad the following expression: 3(4d + 4m + a + 2c)
- c. Was she correct? Explain why or why not.
- d. What is the cost of the bill if a drink costs \$3, a meal costs \$20, an appetizer costs \$5.50, and a slice of cake costs \$3.75?
- e. Suppose the family had a 10% discount coupon for the entire check and then left a 18% tip. What is the total?
- 2. Sally designs web pages for customers. She charges \$135.50 per web page, however she must pay a monthly rental fee of \$650 for her office. Write an expression to determine her takehome pay after expenses. If Sally designed 5 web pages last month, what was her takehome pay after expenses?
- 3. While shopping, Megan and her friend Rylie find a pair of boots on sale for 25% off of the original price. Megan calculates the final cost of the boots by first deducting the 25% and then adding the 6% sales tax. Rylie thinks Megan will pay less if she pays the 6% sales tax first and then takes the 25% discount.
  - a. Write an expression to represent each girl's scenario if the original price of the boots was x dollars.
  - b. Evaluate each expression if the boots originally cost \$200.
  - c. Who was right? Explain how you know.
  - d. Explain how both girls' expressions are equivalent.

#### Lesson 20 Problem Set

#### **Lesson Summary**

- Calculations with rational numbers are used when recording investment transactions.
- Deposits are added to an account balance; money is deposited into the account.
- Gains are added to an account balance, as they are positive returns on the investment.
- Withdrawals are subtracted from an account balance; money is taken out of the account.
- Losses are subtracted from an account balance; as they are negative returns on the investment.
- Fees are subtracted from an account balance; as the bank/financial company is charging you for a service.
- 1. You are planning a fundraiser for your student council. The fundraiser is a Glow in the Dark Dance. Solve each entry below and complete the transaction log to determine the ending balance in the student account.
  - a. The cost of admission to the dance is \$7 per person. Write an expression to represent the total amount of money collected for admission. Evaluate the expression if 250 people attended the dance.
  - b. The following expenses were necessary for the dance, and checks were written to each company.
  - DJ for the dance "Music Madness DJ" costs \$200
  - Glow Sticks for "Glow World Inc." for the first 100 entrants. Cost of glow sticks were \$0.75 each plus 8% sales tax.

DATE	DESCRIPTION OF TRANSACTION	TRANSACTION PAYMENT		BALANCE
	Beginning Balance			1,243.56

Complete the transaction log below based on this information

c. Write a numerical expression to determine the cost of the glow sticks.

Analyze the results.

d. Write an algebraic expression to represent the profit earned from the fundraiser. (Profit is the amount of money collected in admissions minus all expenses.)

- e. Evaluate the expression to determine the profit if 250 people attended the dance. Use the variable p to represent the number of people attending the dance (from part (a)).
- f. Using the transaction log above, what was the amount of the profit earned?

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2. The register below shows a series of transactions made to an investment account. Vinnie and Anthony both completed the register in hopes of finding the beginning balance. As you can see, they do not get the same answer. Who was correct? What mistake did the other person make? What was the monthly gain or loss?

# **Original Register**

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance			
3/1/11	Broker's Fee	250.00		
3/10/11	Loan Withdrawal	895.22		
3/15/11	Refund – Misc. Fee		50.00	
3/31/11	Investment Results		2,012.22	18,917.00

# Vinnie's Work

DATE	DESCRIPTION OF TRANSACTION	PAYMENT	DEPOSIT	BALANCE
	Beginning Balance			18000
3/1/11	Broker's Fee	250.00	:	17750
3/10/11	Loan Withdrawal	895.22		16854.78
3/15/11	Refund – Misc Fee		50.00	1690478
3/31/11	Investment Results		2,012.22	18,917

Anthony's Work

### Lesson 21 Problem Set

#### **Lesson Summary**

- If a number sentence is true, a = b, and you add or subtract the same number from both sides of the equation, then the resulting number sentence will be true.
- If a number sentence is true, a = b, and you multiply or divide both sides of the equation by the same number, then the resulting number sentence will be true.
- 1. Evaluate the following numerical expressions

a.	2 + (-3) + 7	b.	-4 - 1
c.	$-\frac{5}{2} \times 2$	d.	$-10 \div 2 + 3$
e.	$\left(\frac{1}{2}\right)(8) + 2$	f.	3 + (-4) - 1

- 2. Which expressions from Exercise 1 are equal?
- 3. If 3 is divided to two of the equivalent expressions from Exercise 1, write an if-then statement using the properties of equality.
- 4. Write an if-then statement if is multiplied to the following equation: -1 3 = -4
- 5.

Simplify the expression:

5 + 6 - 5 + 4 + 7 - 3 + 6 - 3

Using the expression, write an equation:

Rewrite the equation if 5 is added to both expressions:

Write an if-then statement using the properties of equality.

#### Lesson 22 Problem Set

#### **Lesson Summary**

We work backwards to solve an algebraic equation. For example, to find the value of the variable in the equation 6x - 8 = 40:

- 1. Use the Addition Property of Equality to add the opposite of -8 to each side of the equation to arrive at 6x 8 + 8 = 40 + 8.
- 2. Use the Additive Inverse Property to show that -8 + 8 = 0 and so 6x + 0 = 48.
- 3. Use the Additive Identity Property to arrive at 6x = 48.
- 4. Then use the Multiplication Property of Equality to multiply both sides of the equation by  $\frac{1}{6}$  to get:

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\left(\frac{1}{6}\right)6x = \left(\frac{1}{6}\right)48.
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- 5. Then use the Multiplicative Inverse Property to show that  $\frac{1}{6}(6) = 1$  and so 1x = 8.
- 6. Use the Multiplicative Identity Property to arrive at x = 8.

For each problem below, explain the steps in finding the value of the variable. Then find the value of the variable, showing each step. Write If-then statements to justify each step in solving the equation.

- 1. 7(m + 5) = 21
- 1. -2v + 9 = 25
- 2.  $\frac{1}{3}y 18 = 2$
- 3. 6 + 8p = 38
- 4. 15 = 5k 13

# Lesson 23 Problem Set

# **Lesson Summary**

Equations are useful to model and solve real-world problems. The steps taken to solve an algebraic equation are the same steps used in an arithmetic solution.

For Exercises 1–4, solve each equation algebraically and justify your steps.

- 1.  $\frac{2}{3}x 4 = 20$
- 2.  $4 = \frac{-1+x}{2}$
- 3. 12(x+9) = -108
- 4. 5x + 14 = -7

For Exercises 5–7, write an equation to represent each word problem. Solve the equation showing the steps and then state the value of the variable in the context of the situation.

- A plumber has a very long piece of pipe that is used to run city water parallel to a major roadway. The pipe is cut into two sections. One section of pipe is 12 ft. shorter than the other. If <sup>3</sup>/<sub>4</sub> of the length of the shorter pipe is 120 ft., how long is the longer piece of the pipe?
- 6. Bob's monthly phone bill is made up of a \$10 fee plus \$0.05 per minute. Bob's phone bill for July was \$22. Write an equation to model the situation, using m to represent the number of minutes. Solve the equation to determine the number of phone minutes Bob used in July.
- 7. Kym switched cell phone plans. She signed up for a new plan that will save her \$3.50 per month compared to her old cell phone plan. The cost of the new phone plan for an entire year is \$294. How much did Kym pay per month under her old phone plan?